How to write good TMAs*
(and make your tutor happy)

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1 How to Write Mathematics

Mathematicians write out their work in a particular style, designed for clarity. Students of pure mathematics have to learn to write out their work in this style.

1.1 Why Does It Matter?

Writing out mathematics properly is important because:

- you want the reader of your work to understand what you have done;

- when you use your TMA solutions to help in revision in the future, you may well have forgotten your reasoning; your written solutions should explain the reasoning clearly;

- writing out mathematics properly helps you to think clearly; vague and illogical reasoning doesn't work when you try to write it out in a clear mathematical style.

1.2 Some Guidelines

The main idea behind these guidelines is that it should be easy to understand what you are doing. The reader should never have to wonder "what does this mean?" or "where did this line come from?" or "why is the author doing this?".

1. Always justify each step of your argument. If a statement follows immediately from the line above, say "therefore ....", or use the $\therefore$ sign. If a statement doesn't follow immediately from the line above, briefly justify it. For example, "since G is abelian, $aba^{-1} = aa^{-1}b = \ldots$" or "by de Moivre's Theorem, $z^4 = r^4(\cos 4\theta + \ldots$".

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*Written by a number of M208 tutors for the benefit of all M208 students

2. Always write proper sentences. A sentence is a statement with a clear meaning. A string of symbols, by itself, does not have a clear meaning.

3. An example of the previous point: if you write the string of symbols "$f(x) > 0$" it may not be clear whether you are claiming to have proved $f(x) > 0$, or are restating a fact given in the question, or just writing out a statement which you are going to try to prove.

   So you should say "Therefore $f(x) > 0$" or "We are given $f(x) > 0$" or "We will show that $f(x) > 0$".

4. Do not write strings of equations, with no words linking them, *unless* you make it clear at the start what you're doing. Points 1 to 3 above explain why.

5. When you introduce a new symbol, always say what it stands for. Example: if the letter k has not yet been introduced, don't just say "$x = 2k+1$" without saying what k is. Say "since x is odd, $x = 2k+1$ for some integer k".

6. At the start of a piece of reasoning, say what you are going to do. For example: "To show that function f is onto, take x in the codomain and ... " Don't leave the reader in the dark, wondering where s/he is going.

7. If you are asked to prove that $x \in S$ (say), don't just write "$x \in S$" at the start of your proof. Write "We will show that $x \in S$". That makes it clear that you are not, at this stage, asserting that x is in S. (Failing to follow this guideline can easily lead to circular reasoning.)

8. When you have reached a conclusion, state it, so the reader knows what has been achieved. For example, after showing that for any $(x', y') \in \mathbb{R}^2$ there exists $(x, y) \in \mathbb{R}^2$ such that $g(x, y) = (x', y')$, say "It follows that g is onto."

9. Try to be as accurate as you can using exact values where possible. Thus $\frac{1}{3}$ is better than $0.3333\ldots$ and $\frac{1}{\sqrt{2}}$ is better than $0.414\ldots$ or $.414\ldots$ where the decimal point can get lost. Decimals are rarely required and you can't take a calculator into the exam.

10. You can never, ever, *ever* divide by 0. So if you find you have a 0 on the bottom of a fraction then you have gone wrong!

11. M208 has the convention that, if the domain of a real function is not given, the domain is taken to be the set of all real numbers for which the given rule is applicable (see Book A pages 282 and 301).

    Guideline 10 says that you cannot divide by zero, so points where this happens would be excluded from the domain.

But whenever you divide by something, you need to be *sure* that it's not zero.

For example, rather than saying "Let $f(t) = \dfrac{\cos(t)}{t}$ ", it might be a good idea to say

either   "Let $f(t) = \dfrac{\cos(t)}{t}$ for $t \neq 0$ "

or   "Let $f(t) = \dfrac{\cos(t)}{t}$, where we know $t \neq 0$ because ... "

and that makes it clear that you haven't divided by 0. It may also help you remember, when doing further calculations, which numbers are excluded from the domain.

12. Infinity ∞ is not a number so you cannot divide, multiply or do any other arithmetic operation with ∞. For example, $n \to \infty$ just means that n is getting larger and larger, whereas $n = \infty$ makes no sense. The Analysis books show you how to deal with the concept of infinity rigorously.

13. Study the way things are written out in the M208 units - particularly the Worked Exercises. They are good models of how your solutions should be written out.

14. The comments your tutor makes in your TMAs are there to help you understand the subject and improve future TMAs. Please do spend time reading and absorbing them; there's nothing worse for your tutor than seeing you repeat previous errors.

2   Some things to watch out for

This section illustrates some of the pitfalls you might fall into; beware!

2.1   Assuming what you are trying to prove

You can obviously prove *anything* (whether true or false) if you start by assuming it. Circular reasoning is completely worthless. It's easy to do it by mistake if you don't follow guideline 7 above.

2.2   Arguing from "$x = x$"

If your working ends up with an obviously true statement (like $0 = 0$ or $x = x$), YOU CANNOT DEDUCE ANYTHING from it. Circular reasoning often produces this kind of thing. If you're not clear about this, reread section 2.1 of Unit A3.

Here is a simple example showing how wrong you can be if you try to argue from $x = x$ (see Exercise A112 and Example on pages 216-217 of Book A).

$$-1 = 1 \implies (-1)^2 = 1^2 \implies 1 = 1$$

Of course $1 = 1$ is true but you obviously can't conclude that $-1 = 1$.

For example, suppose you are trying to prove that in the group $(\mathbb{Q}^+, \circ)$ the binary operation $\circ$ is associative, where $a \circ b = \frac{1}{2}ab$ (Book B Exercise B20 pages 46 and 97). You should calculate $a \circ (b \circ c)$ and $(a \circ b) \circ c$ *separately* and show that they have the same value, as is done in B1. Please *don't* write this:

$$a \circ (b \circ c) = (a \circ b) \circ c$$
$$a \circ \tfrac{1}{2}bc = \left(\tfrac{1}{2}ab\right) \circ c$$
$$\tfrac{1}{2}a \left(\tfrac{1}{2}bc\right) = \tfrac{1}{2}\left(\tfrac{1}{2}ab\right)c$$
$$\tfrac{1}{4}abc = \tfrac{1}{4}abc$$

2.3 Be Careful About Notation!

Read what you have written, think about the meaning of the symbols, and make sure that it means what you want it to mean.

a) In the context of functions $\mathbb{R} \to \mathbb{R}$, f is a function, and $f(x)$ is a number. So the equation $f = x^2$ is impossible: a function cannot *equal* a number. It's a sloppy shorthand for $f(x) = x^2$.

b) Use the equals sign properly. An example of how not to do it:

$$\text{the derivative of } x^3 = 3x^2$$

This is obviously nonsense; x^3 does not equal $3x^2$. Instead, write

$$\text{the derivative of } x^3 \text{ is } 3x^2$$

c) In mathematical induction, the notation $P(k)$ is often used to stand for a statement. Then equations like $P(k) = k(k+1)/2$ are nonsense: a statement is a quite different thing from a number, and they cannot be equal.

d) Don't use $=$ when you mean "therefore". Here's an example:

$$\sqrt{2x + 3} = x$$
$$= \quad 2x + 3 = x^2$$

The second $=$ sign here says that $x = 2x + 3$, which is certainly not true, and is not what the author meant.

2.4 Using the "Implies" sign $\implies$

Be careful when using the symbol $\implies$. It does not mean "therefore".

The statement "$A \implies B$" means that *if* A were true, then B would be true also. But it does not mean that either of them actually *is* true.

For example, if A is the statement "the prince is a rhinoceros" and B is the statement "the prince has an extremely thick skin", then the statement $A \implies B$ is true. All rhinos have extremely thick skin.

On the other hand, "A therefore B" says that A is true, therefore B is also true.

2.5 Using the "If and Only if" sign $\iff$

$A \iff B$ means A is true if and only if B is (if you're not clear about "if and only if", see p.203-4 of Book A).

Here is an example of when to use $\iff$. Suppose you want to find the equivalence class $[\![x]\!]$ of x. Then it's not sufficient to say:

if $y \sim x$, then ... string of reasoning ... therefore y is an odd number.

That tells you that every member of $[\![x]\!]$ is odd, but it doesn't follow that every odd number is in $[\![x]\!]$. What you need is this:

$$y \sim x \iff \ldots \cdots \iff y \text{ is odd.}$$

That tells you that $y \sim x$ if and only if y is odd, so $[\![x]\!]$ is the set of all odd numbers.

2.6 Read the Question!

Students often lose marks by not reading the question carefully. Questions are worded quite precisely, and you must do exactly what they ask for.

For example, suppose a question says "Prove X and hence show that Y". The word *hence* means that you must use X in your proof of Y; otherwise you will lose marks. And if a question says "Use mathematical induction to prove ..." then you won't get the marks unless you use induction.

3   More Advice on the Web

A very useful couple of chapters from the book How to Think Like a Mathematician on how to study mathematics at this level is available here:

How to Write Mathematics

http://www1.maths.leeds.ac.uk/˜khouston/pdf/htwm.pdf

It includes advice on writing and reading mathematics, understanding proofs, solving problems, etc.

You may well find the early parts of Mathematical Writing: An undergraduate course by Franco Vivaldi useful.

http://www.thinkingwriting.qmul.ac.uk/sites/default/files/shared/
files/Mathematical%20Writing%20for%20undergraduates%20Vivaldi.PDF

Another guide, particularly useful for writing TMAs, is A Guide to Writing Mathematics by Dr. Kevin P. Lee, Purdue University

www.cs.ucdavis.edu/˜amenta/w10/writingman.pdf

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